Math 2263

Quiz 6

Name   Section   Score

(5 points) 1. Evaluate the double integral by first identifying it as the volume of a solid.

$$\iint_R (6 - x)dA, \ R = \{(x, y)\}|0 \le x \le 6, 0 \le y \le 5\}$$

Solution: $z = 6 - x \ge 0$ for $0 \le x \le 6$, we can interpret the integral as the volume of the solid S that lies below the plan $z = 6 - x$ and above the rectangle $[0, 6] \times [0, 5]$. S is a triangular cylinder whose volume is 5. Area of triangle is $\frac{1}{2} \cdot 6 \cdot 6 = 18$. Thus $\iint_R(6 - x)dA = 80$.

(8 points) Calculate the iterated integral.

$$\int_0^1 \int_0^1 xy\sqrt{x^2 + y^2}dxdy.$$

Solution: Set $u = x^2 + y^2$. So, $du = 2xdx$.

$$\int_0^1 \int_0^1 xy\sqrt{x^2 + y^2}dxdy = \int_0^1 y \int_{y^2}^{1+y^2} \sqrt{u}\frac{du}{2}dy$$

$$= \frac{1}{2}\int_0^1 y \cdot \frac{u^{3/2}}{\frac{3}{2}}\bigg|_{y^2}^{1+y^2} dy$$

$$= \frac{1}{3}\int_0^1 y(1 + y^2)^{3/2}dy - \frac{1}{3}\int_0^1 y^4 dy$$

$$= \frac{1}{6}\int_1^2 w^{3/2}dw - \frac{1}{3}\frac{y^5}{5}\bigg|_0^1 \qquad [\text{ set } w = y^2 + 1, dw = 2ydy]$$

$$= \frac{1}{6}\frac{w^{5/2}}{\frac{5}{2}}\bigg|_1^2 - \frac{1}{15}$$

$$= \frac{1}{15}(2^{5/2} - 1) - \frac{1}{15} = \frac{1}{15}(2^{5/2} - 2) = \frac{2}{15}(2\sqrt{2} - 1).$$

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